Preference Trade-Offs in Material Selectionf or Plumbing Systems

G.V. Loganathan[1], Juneseok Lee[1], D. Bosch[2], S. Dwyer[3], and E. Kleczyk[2]
[1]Department of Civil and Environmental Engineering
[2]Department of Agricultural and Applied Economics
[3]Institute for Community Health
Virginia Tech, Blacksburg, VA 24061
Phone: (540)231-6211, (540)231-3629
Email: gvlogan@vt.edu, jsl0202@vt.edu

Abstract
About 90% of the domestic drinking water plumbing systems in the USA use copper pipes. Recently, there has been a significant increase in pinhole leaks in these copper plumbing pipes. For majority of homeowners, their home is the most valuable asset. The possibility of falling home value and health concerns have caused considerable anxiety among the affected homeowners. There is a need for a decision tool for determining whether to continue to repair or replace the system and which material to use. Currently, PEX (Cross-linked polyethylene) and CPVC (Chlorinated Polyvinyl Chloride) are used along with copper. Stainless steel is being considered but does not have a significant market share. The material selection process is formalized within the frame work of the Analytic Hierarchy Process (AHP). Multiple attributes including price, corrosion resistance, fire retardance, health effects, longevity (cold and hot water), re-sale value of home, and taste and odor are considered. The methodology was implemented with the help of a focus group. The results of the focus group study are reported in this paper.
Keywords: AHP (Analytical Hierarchical Process), preference tradeoffs, plumbing, copper pipe, Water distribution systems

1 - Introduction

In domestic copper plumbing pipes, there has been a noticeable increase of pinhole leaks. Also, there is a concern regarding the behavior of plastic pipes (PEX, CPVC) with respect to strength, fire hazard, final disposal, reaction to chlorine and health effects. Stainless steel is considered less susceptible to corrosion, however, its price is generally considered high for home plumbing compared to the other materials. According to Marshutz' survey (2000), copper accounts for 90% of new homes, followed by PEX (cross linked polyethylene) at 7%, and CPVC (chlorinated polyvinyl chloride) at 2%. Telephone surveys of plumbers tend to show an increased use of plastic pipes due to easier handling in installation and lower material cost. When building a new house or replacing an existing plumbing system, material choice involves cost (material cost plus labor and installation cost), health effects, water quality including microbial growth, corrosion susceptibility, strength, consumer preference from the point of view of selling the house, and behavior in the case of a fire. Figure 1 contains the factors to be considered.

2- Analytical Hierarchy Process (AHP)

The material selection process is formalized with the aid of the Analytical Hierarchy Process (AHP). The AHP is a procedure for assessing preferences over a set of competing alternatives by using pair-wise preferences. It is generally taken that assessing pair-wise preferences is easier as it enables to concentrate judgment on taking a pair of elements and compare them on a single property without thinking about other properties or elements [Saaty (1990)]. It is noted that elicited preferences may be based on the standards already established in memory through a person's experience or education. All rely heavily on measurements and tradeoffs of intangibles in a multi-criteria process. Based on Saaty (1980) the following steps are adopted in performing the analytical hierarchy process.

Step 0: A set of attributes is identified. For plumbing material selection the following seven attributes are considered. Price—includes cost of materials and labor for installation and repair; Corrosion resistance—dependability of material to remain free of corrosion; Fire retardance—ability of material to remain functional at high temperatures and not to cause additional dangers such as toxic fumes; Health effects—ability of material to remain inert in delivering water without threatening human health; Longevity—length of time material remains functional; Resale value of home—people's preference for a particular material including aesthetics; and Taste and odor—ability of material to deliver water without imparting odor or taste (see Figure 1).

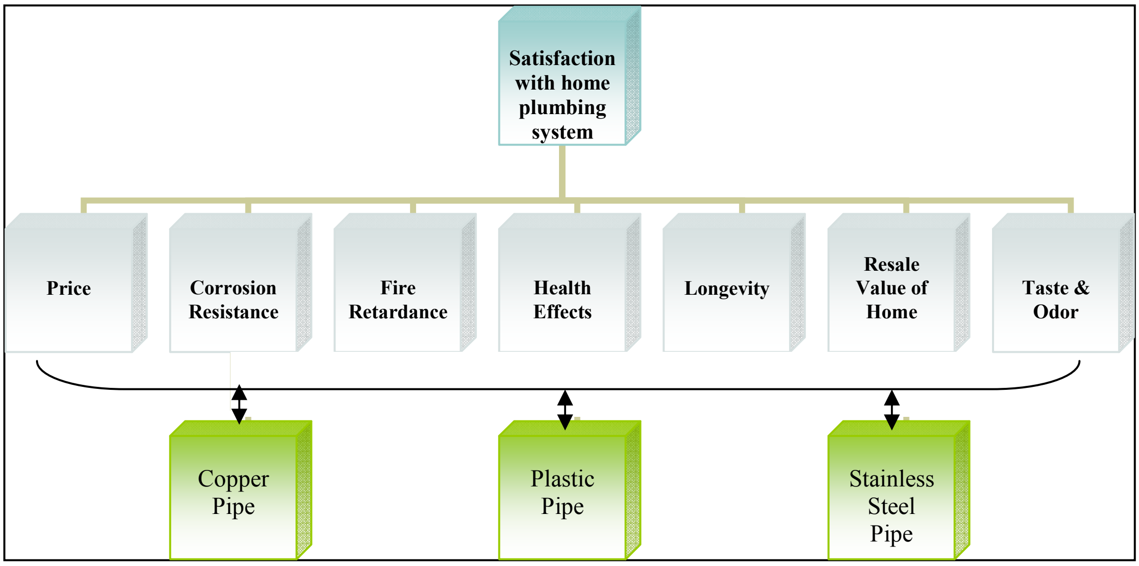

Figure 1 Plumbing Material Selection

Step 1 [Use the standard preference table]: A scale (1-9) of pair-wise preference weights is given in Table 1 (Saaty, 1980).

Table 1. Standard numerical scores

Preference Level	Numerical score, a(i,j) 1-9 scale
Equally preferred	1
Equally to moderately preferred	2
Moderately preferred	3
Moderately to strongly preferred	4
Strongly preferred	5
Strongly to very strongly preferred	6
Very strongly preferred	7
Very strongly to extremely preferred	8
Extremely preferred	9

Step 2 [Develop the pair-wise preference matrix]: As shown in Tables 2 and 3, each participant is asked to fill in a 7x7 attribute matrix of pair-wise preferential weights (Table 3).

Table 2. Pair-wise preference weight matrix

	Attribute 1	Attribute 2	…	Attribute n
Attribute 1	w1/w1	w1/w2	…	w1/wn
Attribute 2	w2/w1	w2/w2	…	w2/wn
…	…	…	…	…
Attribute n	wn/w1	wn/w2	…	wn/wn
Sum	X/w1	X/w2	…	X/wn

in which: X = (w1+w2+…+ wn).
From the pair-wise preference matrix, we observe the following.

$$\begin{bmatrix} w1/w1 & w1/w2 & ... & w1/wn \\ w2/w1 & w2/w2 & ... & w2/wn \\ ... & ... & ... & ... \\ wn/w1 & wn/w2 & ... & wn/wn \end{bmatrix} \begin{Bmatrix} w1 \\ w2 \\ ... \\ wn \end{Bmatrix} = n \begin{Bmatrix} w1 \\ w2 \\ ... \\ wn \end{Bmatrix}$$

in which: $\begin{Bmatrix} w1 \\ w2 \\ ... \\ wn \end{Bmatrix}$ = {w} = required global preference vector (of weights)

Table 2 is a reciprocal matrix in that the off diagonal elements are reciprocals of each other, Numerical score, a(i,j) = $\frac{\text{weight for criterion, i, wi}}{\text{weight for criterion, j, wj}}$ = 1/a(j,i)

Table 3. Pair-wise preference weight matrix

	P	C	F	H	L	R	T
P	1	0.20	0.25	0.14	0.20	0.50	0.33
C	5	1	5	0.33	1	6	1
F	4	0.20	1	0.17	0.33	1	0.25
H	7	3	6	1	4	8	3
L	5	1	3	0.25	1	2	0.33
R	2	0.17	1	0.13	0.50	1	0.50
T	3	1	4	0.33	3	2	1
Sum	27.00	6.57	20.25	2.35	10.03	20.50	6.42

(P: price, C: corrosion resistance, F: fire retardance, H: health effects, L: longevity, R: resale value of home, and T: taste and odor)

In row H and column P, the entry of 7 implies that health effects are very strongly preferred in comparison to price in the ration of 7:1. In fact, row H for health effects overwhelms all other attributes with the entries staying well above 1. In row P and column C, the cell value of 0.2 indicates corrosion resistance is strongly preferred to price in the ratio of 5:1.

Step 3[Evaluate the re-scaled pair-wise preference matrix]: A rescaled preference matrix is generated by dividing each column entry in Table 3 by that column's sum yielding Table 4. The last column "Average" contains average values for each row and shows the ranking of the attributes. Table 4 shows the ordered relative ranking of the attributes.

Table 4. Rescaled pair-wise weight matrix

Attribute	P	C	F	H	L	R	T	Average
P	0.04	0.03	0.01	0.06	0.02	0.02	0.05	0.03
C	0.19	0.15	0.25	0.14	0.10	0.29	0.16	0.18
F	0.15	0.03	0.05	0.07	0.03	0.05	0.04	0.06
H	0.26	0.46	0.30	0.43	0.40	0.39	0.47	0.38
L	0.19	0.15	0.15	0.11	0.10	0.10	0.05	0.12
R	0.07	0.03	0.05	0.05	0.05	0.05	0.08	0.05
T	0.11	0.15	0.20	0.14	0.30	0.10	0.16	0.17
Sum	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 5. Relative ranking of attributes

Attribute	Weight
Health Effects	0.38
Corrosion resistance	0.18
Taste and odor	0.17
Longevity	0.12
Fire resistance	0.06
Resale value	0.05
Price	0.03

Step 4: For each attribute, pair-wise weight matrix and the associated rescaled matrix for the three pipe materials are obtained. Results for price attribute are shown as Table 6. These procedures are iterated for other attributes.

Table 6. Pair-wise matrix and the associated rescaled matrix for the attribute Price

Price								
Material	Mat. A	Mat. B	Mat. C		Mat. A	Mat. B	Mat. C	Average
Mat. A	1.000	0.333	2.000	Mat. A	0.222	0.200	0.333	0.252
Mat. B	3.000	1.000	3.000	Mat. B	0.667	0.600	0.500	0.589
Mat. C	0.500	0.333	1.000	Mat. C	0.111	0.200	0.167	0.159
Sum	4.500	1.667	6.000	Sum	1.000	1.000	1.000	1.000

We obtain the final ranking of the 3 materials by using the average ranking for the three materials for the 7- attributes namely, price, corrosion resistance, fire retardance, health effects, longevity, resale value of home, taste and odor and the average ranking of the criteria themselves from Table 4. The respective matrix and the vector are given in Table 7.

Table 7. average ranking for the materials and criteria

	P	C	F	H	L	R	T
Mat. A	0.252	0.164	0.444	0.328	0.250	0.297	0.250
Mat. B	0.589	0.297	0.111	0.261	0.250	0.164	0.250
Mat. C	0.159	0.539	0.444	0.411	0.500	0.539	0.500

Average
0.03
0.18
0.06
0.38
0.12
0.05
0.17

Multiplying the above pipe material preference matrix and the attribute preference vector yields Table 8 shown below.

Table 8. Final preference matrix

Material	Preference
Mat. A	0.279
Mat. B	0.261
Mat. C	0.460

Based on the highest relative preference score of 0.460, Material C obtains a rank of 1, followed by Material A for a rank of 2, and C with a rank of 3. The consistency check for the material related matrices and average relative criteria preference vector are performed following Saaty (1980) as given in Step 5. Participants were advised to reassess the pair-wise weights if consistency check fails.

Step 5[Perform consistency check]: From step 2, we note that the calculated maximum eigen value, $\lambda max = n$. If it is different from n, we have inconsistencies in our weight assignments. Saaty (1980) defines a consistency index as C.I. = $(\lambda max - n)/(n-1)$. He suggests that if the ratio of C.I. to R.I. (random index given in Table 2) is less than 0.1 [i.e. C.I./R.I. < 0.1], the weights should be taken as consistent. Table 9 contains the random index values calculated from randomly generated weights as a function of the pair-wise matrix size (number of criteria).

Table 9. Random Index (R.I.)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	0	0.58	0.90	1.12	1.24	1.32	1.41	1.45	1.49	1.51	1.48	1.56	1.57	1.59

However, we adopted the *simplified procedure* of using the rescaled matrix as illustrated in the above. We restate the pair-wise preference matrix from Table 2 for convenience

	Attribute 1	Attribute 2	…	Attribute n
Attribute 1	w1/w1	w1/w2	…	w1/wn
Attribute 2	w2/w1	w2/w2	…	w2/wn
…	…	…	…	…
Attribute n	wn/w1	wn/w2	…	wn/wn
Sum	(w1+w2+…+ wn)/w1	(w1+w2+…+ wn)/w2	…	(w1+w2+…+wn)/wn

to obtain the following [Anorm] in Table 10.

Table 10. Rescaled pair-wise preference matrix [Anorm]

	Attribute 1	Attribute 2	…	Attribute n	Average
Attribute 1	w1/X	w1/X	…	w1/X	Ave(w1/X)
Attribute 2	w2/X	w2/X	…	w2/X	Ave(w2/X)
…	…	…	…	…	…
Attribute n	wn/X	wn/X	…	wn/X	Ave(wn/X)
Sum	1	1		1	

in which: X = (w1+w2 + … + wn). If the weights are consistent all columns of [Anorm] should be identical. Therefore, it verifies the consistency of the assessments. Also, note the rescaled columns sum to be 1 forming a stochastic matrix in the sense probabilities sum to be 1. Such a matrix has a maximum eigenvalue of 1. The last column of Table 10 is the relative preference vector between criteria 1, 2, and n obtained by averaging the columns of the rescaled pair-wise matrix. Multiply Step 2 pair-wise preference weight matrix with the relative preference criteria vector from the last column of Table 10 as shown below.

Attribute 1	Attribute 2	…	Attribute n
w1/w1	w1/w2	…	w1/wn
w2/w1	w2/w2	…	w2/wn
…	…	…	…
wn/w1	wn/w2	…	wn/wn

Average
Ave(w1/X)
Ave(w2/X)
…
Ave(wn/X)

Product
[(w1/w1)Ave(w1/X)+(w1/w2) Ave(w2/X)+…+(w1/wn) Ave(wn/X)]
[(w2/w1)Ave(w1/X)+(w2/w2) Ave(w2/X)+…+(w2/wn) Ave(wn/X)]
…
[(wn/w1)Ave(w1/X)+(wn/w2) Ave(w2/X)+…+(wn/wn) Ave(wn/X)]

Divide each entry of the product column by the respective entry of the average column to yield

Ratio = Product/Average
[(w1/w1)Ave(w1/X)+(w1/w2) Ave(w2/X)+…+(w1/wn) Ave(wn/X)]/ Ave(w1/X)
[(w2/w1)Ave(w1/X)+(w2/w2) Ave(w2/X)+…+(w2/wn) Ave(wn/X)]/ Ave(w2/X)
…
[(wn/w1)Ave(w1/X)+(wn/w2) Ave(w2/X)+…+(wn/wn) Ave(wn/X)]/ Ave(wn/X)
Sum of ratios =

Divide the sum of ratios by n to obtain A.R. average ratio. It should be close to n. The consistency index (C.I.) is [A.R. –n]/(n-1). Check the C.I. against the permissible limit by C.I./R.I. < 0.1 to accept the AHP results. If it fails, the pair-wise weights should be re-assessed.

3 - Results

Table 11 contains the attribute ranking for 10- participants. From Table 11, it is seen that the participants rank health effects the highest, followed by taste and odor, corrosion resistance, and longevity; price, resale value and fire resistance are showing the lower ranks of the list. These results indicate that health, and taste and odor may be a surrogate for the purity of water that dominates preferences for a plumbing material. The final ranking of the materials in Table 12 are part of on going research. We have indicated themas Mat. A, B, and C because the results are only preliminary. The manner and the type of information provided to the participants is important in preference changes.

Table 11. Participant ranking of attributes

Participant/ Attribute	1	2	3	4	5	6	7	8	9	10
P	0.034	0.161	0.025	0.040	0.200	0.063	0.310	0.202	0.048	0.087
C	0.182	0.037	0.107	0.061	0.090	0.091	0.158	0.111	0.112	0.256
F	0.060	0.056	0.078	0.085	0.070	0.119	0.044	0.057	0.188	0.044
H	0.385	0.211	0.395	0.304	0.270	0.450	0.154	0.369	0.188	0.367
L	0.120	0.097	0.101	0.132	0.090	0.120	0.154	0.067	0.112	0.062
R	0.054	0.020	0.027	0.127	0.050	0.098	0.103	0.064	0.033	0.029
T	0.165	0.419	0.267	0.250	0.230	0.059	0.078	0.131	0.318	0.155

(P: price, C: corrosion resistance, F: fire retardance, H: health effects, L: longevity, R: resale value of home, and T: taste and odor)

Table 12. Final ranking of the materials.

	1	2	3	4	5	6	7	8	9	10
Mat. A	0.279	0.209	0.337	0.310	0.201	0.321	0.273	0.218	0.184	0.178
Mat. B	0.261	0.175	0.121	0.191	0.273	0.103	0.371	0.513	0.181	0.383
Mat. C	0.460	0.616	0.540	0.499	0.526	0.576	0.356	0.269	0.634	0.438

4 – Conclusion

In this paper, we have presented a formal methodology for considering various attributes involved in home plumbing. It is clear that health considerations clearly dominate the consumer choice. The mental stress resulting from frequent leaks is also a major issue. While copper remained as a relatively inert carrier of water, the corrosion leaks have forced consumers to consider alternatives including other materials, the use of corrosion inhibitors such as phosphate, and lining the interior of the pipe. The writers are part of a research group that is considering these issues and the results will be reported when more insight is gained.

5 - Acknowledgement

The writers gratefully acknowledge the support provided by the National Science Foundation under the grant DMI-0329474.

6 - References

[1] Copper development association. Copper tube handbook. 2005. http://www.copper.org/resources/pub_list/pdf/copper_tube_handbook.pdf (August 28, 2005)

[2] Loganathan, G.V., Farooqi, O., Lee, J., Batterton, S. 2005. Plumber telephone surveys. Virginia Tech.

[3] Marshutz, S., "Hooked on Copper", Reeves Journal, March 2001. http://www.reevesjournal.com/CDA/ArticleInformation/features/Features_Index/1,3816,27-820,00.html (August 28, 2005)

[4] Personal communication. 2005. Lowe's and Home Depot in Christiansburg, VA. USA.

[5] Plastic Pipe and Fittings Association. Installation handbook: CPVC hot & cold water piping. 2002. http://www.ppfahome.org/pdf/cpvc_manual.pdf (August 28, 2005)

[6] Plastic Pipe and Fittings Association. Installation handbook: PEX hot and cold water-distribution systems. 2002. http://www.ppfahome.org/pdf/PEX%20Installation%20Handbook.pdf (August 28, 2005)

[7] Saaty, T.L. "*The Analytic Hierarchy Process*", McGraw-Hill, 1980, New York.

[8] Saaty, T.L. "How to make a decision: The Analytic Hierarchy Process". *European Journal of Operational Research*, 1990 (48), pp. 9-26.

[9] Winston, W.L., "Operations Research", Duxbury Press, Belmont, 1998, California.

[10] Winston, W.L., "Practical Management Science", Duxbury Press, Wadsworth Publishing Company, 1997, USA

[11] Woodson, R.D., "2003 International Plumbing Code Handbook", McGraw-Hill, 2004, New York.

[12] Woodson, R.D., "Plumber's Standard Handbook", McGraw-Hill, 1999, New York.